



Denoising of Arrhythmia ECG Signals

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ABSTRACT

This study is about using the genetic algorithm (GA) with wavelet transform (WT) for Arrhythmia Electrocardiogram (ECG) signal denoising purposes. The WT is a time-frequency signal analysis, and the GA is an optimization technique based on survival of the best solution using the maximized or minimized fitness value obtained from the fitness function. In this study, the parameters of WT are used as inputs for the GA for denoising the input signal that is corrupted by white Gaussian noise and gives an output of as fitness value. The input corrupted signal will pass through decomposition process to extract approximation and details coefficients, then thresholding the details coefficients using a threshold value in order to remove the noise, and finally reconstruction of the signal using the approximation and denoised details coefficients. The results of denoising ECG Arrhythmia records were compared with other studies in the field of wavelet denoising, and the comparison showed that the results of this work is better.

Keywords: ECG, Cardiac arrhythmia, Wavelet transform, Denoising, Genetic algorithm

INTRODUCTION

Working with data acquired from nature is often accompanied by noise. Additive noise is one type of the different noise types. Noise exists in a high or low signal to noise ratio (SNR) due to many conditions or factors. For this reason, denoising techniques are required to overcome this problem and to reduce the amount of the noise that exists in the signal because the extraction of information from the signal requires a signal with no or very little noise [1]. The ECG is recording of the heart's electric activity of depolarization and repolarization of the atrial and ventricular chambers of the heart. A normal ECG signal consists chiefly of P wave, QRS complex and T wave as in Figure 1. Understanding these various components allows obtaining important knowledge about the function of the patient's heart [2].

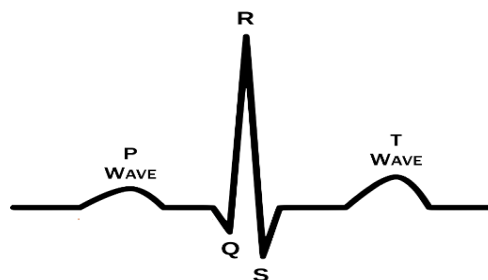


Figure 1 Ideal ECG Section [2]

The noise sources that affect ECG during recording are electrical activity of muscles and instability of electrode skin contact [3]. In signal processing, there are many techniques to perform denoising. However, among all the available techniques, denoising based on WT has proved outstanding performances [4], because WT can be used with non-stationary signals like ECG signal [5]. Wavelet transform threshold shrinkage (WTS) is a very robust method used to remove noise from a signal corrupted by noise [6,7]. Wavelet means 'small wave' with energy concentrated at a specific point [8] and transform means 'representing the signal into another form'. Genetic Algorithm is an optimization technique that is inspired from the living organisms, from how chromosomes and their genes are mates

[9]. GA is based on the survival of the fittest solution from a number of solutions called population [10]. Genetic Algorithm is used to give optimal WT denoising parameters as wavelet filter, decomposition level, scale factor, threshold value and scale factor, by running the WT into a number of iterations and then the optimal denoising parameters will be given depending on the minimum output mean square error MSE_o [5]. The paper is organized as follows: Section 2 includes works related to this paper; section 3 contains an overview of the wavelet transform and the denoising process, Section 4 talks about the genetic algorithm, while section 5 presents the results and their discussion. Finally, sections 6 demonstrate the conclusions.

Related Works

Many researches have presented on ECG signal denoising based on wavelet transform, as the study of El-Dahshan [5] introduces an effective hybrid scheme for the denoising of electrocardiogram (ECG) signals corrupted by non-stationary noises using the GA and DWT. The study of Salih, Amer, and Habib [11], showed a new algorithm for bio medical signals denoising by using genetic algorithm, the coefficients of wavelets filters alter smoothly until the best SNR for output signal achieved. The study of Chitrangi, and Harishchandra [12], showed the selection of best denoising parameters after several experiments. Zaid, Mohammed, Ahmad, and Laith [13] showed a new technique for denoising by using b-hill climbing algorithm with wavelet transform. Experimental results show the validity of the algorithm, and in this research, DWT is used with GA for denoising ECG Arrhythmia records.

Wavelet Denoising of ECG Signal

Signals consist of different features in time and frequency and their high-frequency components would have shorter time duration than their low-frequency components. In order to achieve a good time resolution for high-frequency transients and good frequency resolution for low-frequency components, Morlet in 1982, first introduced the idea of wavelets as a family of functions constructed from translations and dilations of a single function called Mother wavelet and defined by Kaur and Daubechies [14,15]:

$$\psi_{a,b}(t) = \left(\sqrt{|a|}\right)^{-1} \psi\left(a^{-1}(t-b)\right) \quad a, b \in R, a \neq 0 \quad (1)$$

Where a is a scaling or dilation parameter that measures the degree of compression and b is the translation or shift parameter that determines time location of the wavelet [6]. The WT computation requires four types of filters. In the decomposition stage, the fine approximation coefficients $a_{j+1}(k)$ is applied to two analysis filters to compute the coarse approximation and detail coefficients $a_{j+1}(k)$ and $h_a(n)$ respectively. The two analysis filters are: low pass filter (LPF) $h_a(n)$ and high pass filter (HPF) $d_{j+1}(k)$. In the reconstruction stage the coarse wavelet coefficients $d_{j+1}(k)$ and the coarse approximation coefficients $a_j(k)$ are applied to two reconstruction filters to produce the fine approximation coefficients $a_j(k)$. The two reconstruction filters are: LPF $g_s(n)$ and the HPF $g_s(n)$ [16].

Decomposition from Fine Scale to Coarse Scale (DWT)

The decomposition from fine scale to coarse scale means to go from level j to level $j+1$. This means that from a given approximation coefficients $a_{j+1}(k)$ at level j the coefficients $a_{j+1}(k)$ and $d_{j+1}(k)$ at level $j+1$ can be computed [8,11] as in Figure 2.

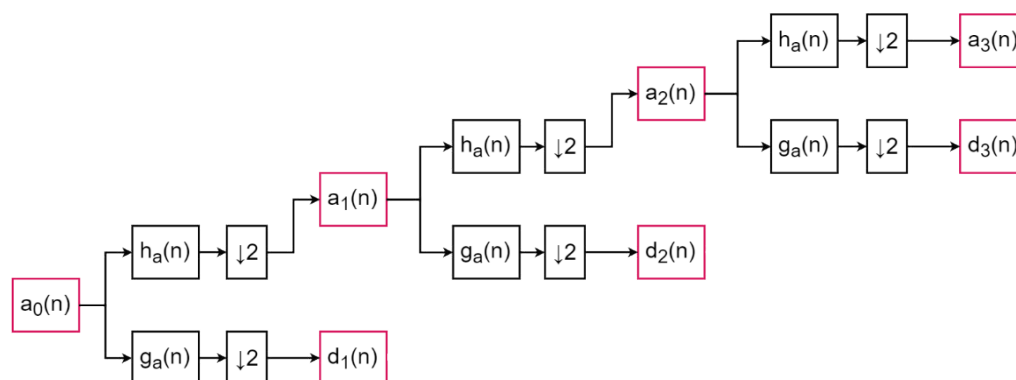


Figure 2 Three levels DWT implementation via analysis filter banks [17]

Reconstruction from Coarse Scale to Fine Scale (IDWT)

The reverse process of combining the coarser approximation and detail coefficients to yield the approximation coefficients at a finer resolution, performed by digital filtering, is referred to as reconstruction or synthesis. The mathematical manipulation that affects synthesis is called the inverse discrete wavelet transform (IDWT) [8,11] as in Figure 3.

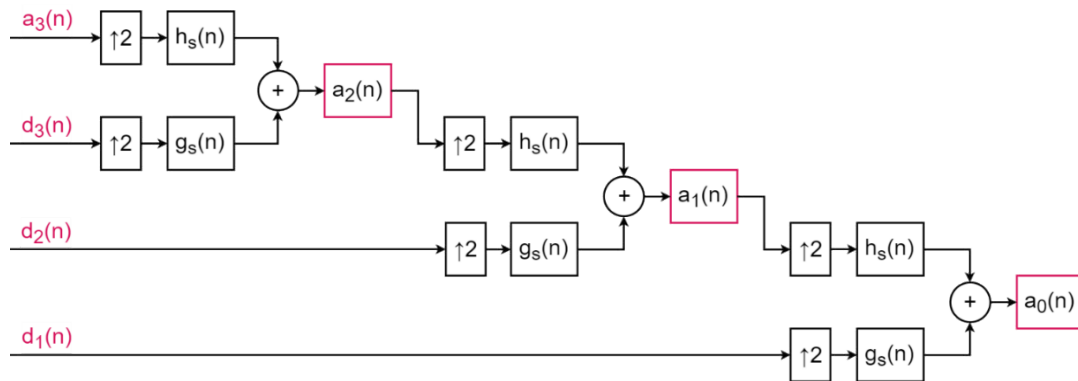


Figure 3 Three levels IDWT implementation via synthesis filter banks [17]

Wavelet Families

There are a large number of wavelet families that can be used for continuous and discrete analysis. For discrete analysis, some famous examples are Daubechies, Symlets, and Coiflets. For continuous analysis, examples include Morlet, Meyer, and Gaussian wavelets. The characteristics of the signal and the nature of application determine which type of wavelet families is suitable for the work, Table 1 shows most of the known wavelet families [5].

Table 1 A list of the known and used wavelets [18]

Family Short Name	Wavelet Family Name
db	Daubechies wavelets
sym	Symlets
coif	Coiflets
bior	Biorthogonal wavelets
rbio	Reverse biorthogonal wavelets
meyr	Meyer wavelet
dmey	Discrete approximation of Meyer wavelet
gaus	Gaussian wavelets
mexh	Mexican hat wavelet
morl	Morlet wavelet
cgau	Complex Gaussian wavelets
shan	Shannon wavelets
fbsp	Frequency B-Spline wavelets
cmor	Complex Morlet wavelets
fk	Fejer-Korovkin wavelets

Denoising

$$s_i = f + \sigma e_i \quad (2)$$

It is the most general One-Dimensional (1D) form of a signal containing noise [19]. f is the original signal. The e_i is Gaussian random variable distributed as $N(0,1)$. The variance of the σe_i is s_i . The s_i is the noisy signal [20]. Denoising steps summarized as:

1. **Decompose:** This includes choosing the wavelet and level J . Then compute the wavelet decomposition of a signal s_i up to J levels [21].

2. **Threshold detail coefficients:** for each level threshold detail and approximation coefficients [21].
3. **Reconstruct:** regenerate the signal using the original approximation coefficients of level J and the modified detail coefficients of levels from 1 to J [21].

Threshold Selection Rules

There are four threshold selecting rules implements in [the select] command in MATLAB toolbox:

Rigrsure: Rigorous sure or shrink sure, it is used for One-Dimensional (1D) data, threshold achieved by minimizing Stein's Unbiased Risk Estimate depends on shrinkage function and the multiresolution level [22].

The sorted squared coefficients are computed as: $S_c = (\text{Sort}(|c|))^2$

Then the threshold value is selected using the square root of the minimum risk as:

$$T_{val} = \sqrt{\min \left(\frac{(N-2A) + \left(\sum_{i=1}^N S_{ci} \right) + (B * S_c)}{N} \right)} \quad (3)$$

where $N = \text{length}(c)$, c is the coefficients, $A = (1 \ 2 \ \dots \ N)$ and $B = (N-1 \ N-2 \ \dots \ 0)$

Sqtwolog: It is a threshold rule proposed by Donoho and Johnstone which is called universal threshold and it is used for One-Dimensional data, regardless of the shrinkage function, as a coefficient of size n with white Gaussian noise distributed at $N(0,1)$ [19,22].

$$T_{val} = \sqrt{2 \log(N)} \quad (4)$$

Heursure: Heuristic sure, which is a mixture of the rigrsure and sqtwolog threshold selection rules, as a result, if the SNR is very small, the heursure estimate is very noisy. So, if such a situation is detected, the sqtwolog threshold is used [6].

$$T_{val} = \begin{cases} \sqrt{2 \log(N)} & \text{if SNR very small} \\ \min(rigrsure(c), \sqrt{2 \log(N)}) & \text{otherwise} \end{cases} \quad (5)$$

Minimaxi: It uses a fixed threshold chosen to yield minimax performance for MSE against an ideal procedure. The minimax estimator is the option that realizes the minimum, over a given set of functions, of the maximum MSE [5].

$$T_{val} = \begin{cases} 0 & N \leq 32 \\ 0.3936 + 0.1829 \times \frac{\log(N)}{\log(2)} & \text{otherwise} \end{cases} \quad (6)$$

Threshold Functions

Soft and hard are examples of threshold selection rules. Figure 4 explains the two methods [19].

Hard: A hard threshold function makes all signal values smaller than T_{val} to zeroes. As a result, there appears discontinuity in the reconstructed signal. The equation can be expressed as [23,24]:

$$T_{fun}(s_i) = \begin{cases} s_i & |s_i| \geq T_{val} \\ 0 & |s_i| < T_{val} \end{cases} \quad (7)$$

Soft: After the determination of T_{val} , a soft threshold function makes signal values smaller than T_{val} zeros and T_{val} is subtracted from the signal values greater than T_{val} as in the equation below [23,24]:

$$T_{fun}(s_i) = \begin{cases} \text{sgn}(s_i) * (|s_i| - T_{val}) & |s_i| \geq T_{val} \\ 0 & |s_i| < T_{val} \end{cases} \quad (8)$$

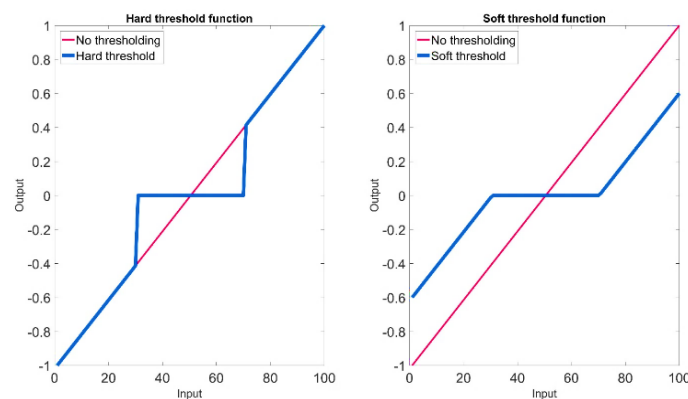


Figure 4 Hard and soft threshold functions [23,24]

Un-Scaled Noise and Non-white Noise

Dealing with unscaled or nonwhite noise can be handled using rescaling output threshold (see SCAL parameter in WDEN), available in MATLAB wavelet toolbox [5], Table 2 gives details about the values of the SCAL parameter.

Table 2 Different scal parameter values and their description [5]

Values	Description
scal = one	Corresponds to the basic model
scal = sln	Handles threshold rescaling using a single estimation of level noise based on the first-level coefficients. When you suspect a nonwhite noise e , thresholds must be rescaled by a level dependent estimation of the level noise. The same kind of strategy as in the previous option is used by estimating σ level by level
scal = mln	Handles threshold rescaling using a level-dependent estimation of the level noise

Genetic Algorithm

GA is an optimization technique that is inspired from the process of natural selection in living organisms. Optimization is the process of adjusting the inputs of a device, mathematical process, or experiment to find the minimum or maximum output [10]. The GA it's an optimization and search technique based on the principles of genetics and natural selection [10]. A GA allows a population composed of many individuals to evolve under specified selection rules to a state that maximizes the fitness (i.e., minimizes the fitness function).

Initialization: GA has a few variables that need to be initialized before proceeding, these variables and their values are the population represents the total number of individuals in the generation which is 50, and the individual length represents the length of an individual which is 14 bits. A crossover probability gives the number of individuals that will be selected for crossover process which is 80%, and the mutation probability determine how many individuals can be subjected to mutation which is 1%, while the elite count represent how many individuals can survive to the next generation without passing through crossover or mutation processes of GA its 5% [5]. The GA also needs to initialize the population by randomly generating [11] binary individuals of 0's and 1's with individual length of N_{bits} . The total number of individuals used in the population N_{pop} is 50. Figure 5 illustrates the population of this work.

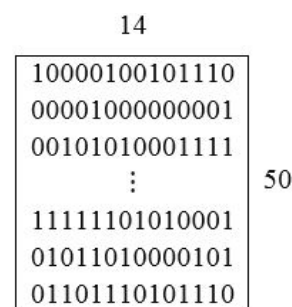


Figure 5 Sample population

Encoding and Decoding: Encoding is the process of representing individual genes. Binary encoding is the most common way used [25], where each individual is encoded with a binary bits string of 0s and 1s [25]. The binary encoding method is used because of its simplicity in computation and implementation in the selection, crossover, and mutation operations [19]. Also, it takes less memory [8]. Each individual is a binary code that represents threshold value, wavelet filter, decomposition level, and scale factor respectively, Table 3 gives more details on the location and the number of bits required for each variable. From Table 3 the wavelet filter needs 7 binary bits to represent all its possible values, decomposition level needs 3 bits to represent its 8 values, and the same thing for threshold value and scale factor; both need 2 binary bits to represent their 4 values.

Table 3 The binary encoding method used with GA

Variable	Range of values	Required bits	Location of bits
Threshold value	sqtwolog, rigrsure, heursure, minimaxi (4 values)	2	1-2
Wavelet filter	db1 ... db45, coif1 ... coif5, sym2 ... sym30, dmey, fk4, ... fk22, bior1.1 ... bior6.8, rbio1.1 ... rbio6.8 (115 values)	7	3-9
Decomposition level	1, 2, 3, 4, 5, 6, 7, 8 (8 values)	3	10-12
Scale factor	one, sln, mln (3 values)	2	13-14

Fitness function: It is the function that gives each individual a value, in which GA uses this value in which GA uses this value for the selection of individuals in the selection process [19]. The fitness function in this work represents wavelet denoising, and the output of the function is the minimal MSE_o . The MSE_o calculates the average of the squares of the errors between the original and the denoised signal [26]. The fitness function in this work is designed to optimize only four parameters as threshold value, wavelet filter, and decomposition level, and gives an output of MSE_o .

Selection: The first process in GA is the selection of individuals from the population, based on the fitness value of each individual obtained from the fitness function. The selection method used is the Stochastic Universal Sampling (SUS) which uses M equally spaced pointers, where M is the number of selections of the individuals required. The step between pointers is P , and the first pointer position is chosen randomly in an interval of $[0, P]$ which is $ptr1$, then add P to $ptr1$ to get the position of $ptr2$ and continue until reaching the last pointer $ptrN$, as illustrated in Figure 6. Below are the main steps of the SUS selection algorithm [9,19]:

1. Set M which is the total number of individuals to keep.
2. Calculate the total fitness of the population as F .
3. Calculate the distance between the pointers as $P=F/M$.
4. Choose a random number R between 0 and P .
5. Define pointers as $R+(i * P)$ where $i = 0 \dots (M - 1)$ [9,19].

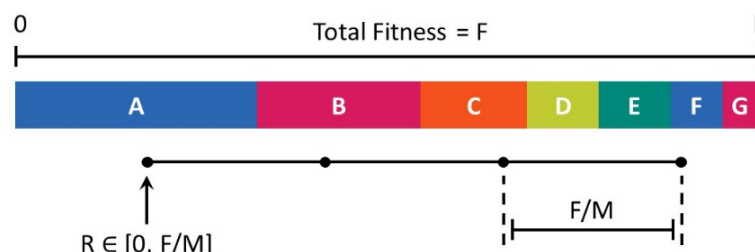


Figure 6 The Stochastic Universal Sampling selection method

Crossover: In the crossover process, two individuals or parents are selected based on their fitness value for crossover, in order to produce two new children or offspring. The crossover function used here is a scattered method, in which a vector is created with a random zero and ones with a length equal to the length of any individual, where one means to exchange the bit but zero means not exchanging it. This random vector is applied to every parent, as illustrated in Figure 7. The crossover rate used in this work is 80% [27].

Parent 1	:	0 1 0 1 1 1 0 0 0
Parent 2	:	1 1 1 1 0 0 0 1 0
Mask	:	1 0 1 0 0 0 0 1 1
Child 1	:	1 1 1 1 1 1 0 1 1
Child 2	:	0 1 0 1 0 0 0 0 1

Figure 7 Scattered crossover process

Mutation: Mutation refers to a random change in the genes of an individual, and usually mutation occurs with very low probability like 1% [16]. A single point mutation changes a 1 to a 0, and vice versa as in Figure 8. Mutation genes are randomly selected from the population. Increasing the number of mutations, the algorithm's freedom to search outside the current region of variable space also increases [27].

Original	0 1 0 1 1 1 0 0 0
Applying mutation	
Mutated	0 1 0 1 0 1 0 0 0

Figure 8 Mutation operation

Stopping conditions: GA has a few conditions. If one of them is satisfied, GA will stop, otherwise, it will continue running [19]:

1. **Maximum generations:** If the specified number of generations have been reached.
2. **Fitness value:** If the desired fitness value is reached.
3. **Stall generations:** If there is no improvement in the fitness value for a sequence of generations.
4. **Function tolerance:** If the change in the fitness value is very small that is less than function tolerance [19].

RESULTS AND DISCUSSION

In this section the results will be presented and discussed. Ten records of ECG signal from MIT-BIH Arrhythmia Database have been selected, each record 1024 samples were taken from [28]. Cardiac arrhythmia is a term for some of a big and heterogeneous number of conditions by which there's abnormal electrical activity in the heart. One's heartbeat might be too quickly or too slow and might be regular or irregular [2]. Two metric terms are used to evaluate the performance of denoising process:

The mean squared error (MSE) calculates the average of the squares of the errors between the original and the denoised signal, whenever the value is near to zero it is better and its used as a fitness value for the fitness function.

$$MSE_o = \frac{1}{N} \sum_{t=1}^N (f(t) - s_o(t))^2 \quad (9)$$

The signal to noise ratio (SNR) is the measure of signal quality, the higher ratio indicates a better performance, and its used to measure the performance of denoising process.

$$SNR_{o(dB)} = 10 \times \log_{10} \left(\frac{\sigma_f^2}{\sigma_{e_o}^2} \right) \quad (10)$$

As stated before ten real records of ECG Arrhythmia signal are used from the MIT-BIH database, the flowchart of the proposed system is illustrated in Figure 9, and the best denoising parameters results obtained from this system are listed in Table 4.

Table 4 The best denoising parameters

SNR Input	Threshold Function	Threshold Value	Wavelet Filter	Synthesis Level	Scale Factor
1	Hard	sqtwolog	sym11	4	one
10	Hard	sqtwolog	rbio6.8	4	mln
20	Hard	sqtwolog	db3	4	one
30	Hard	heursure	db16	4	sln
40	Hard	heursure	db10	4	one
1	Soft	heursure	coif2	7	one

10	Soft	rigsure	db7	6	one
20	Soft	rigsure	db5	7	one
30	Soft	rigsure	db39	4	one
40	Soft	heursure	db38	3	one

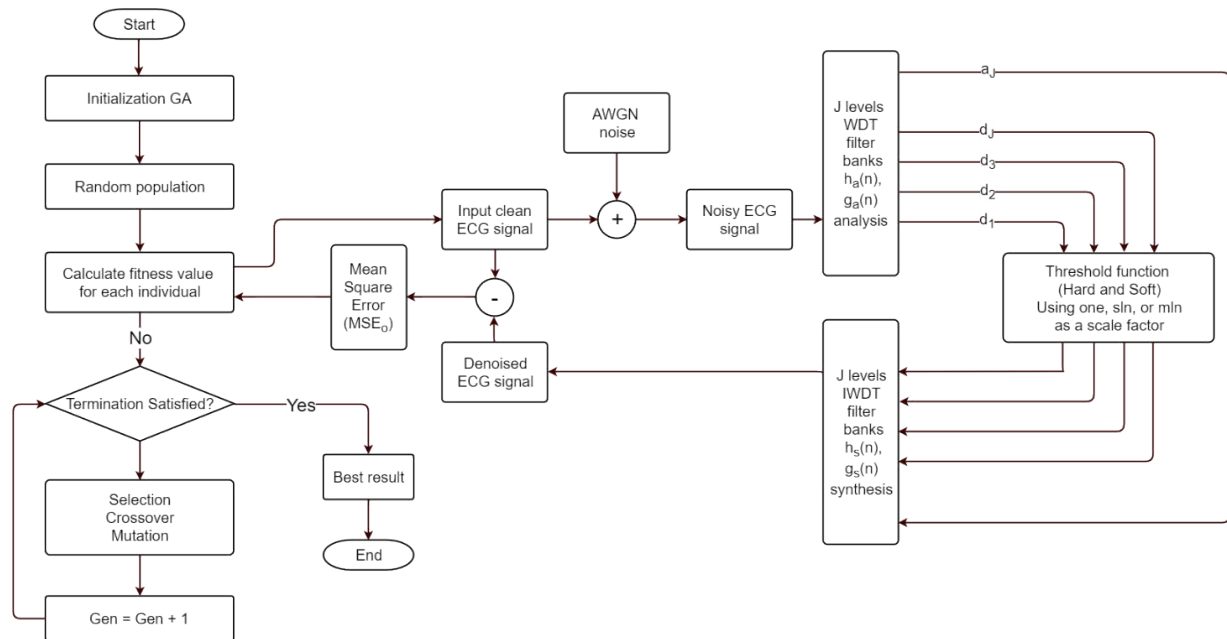


Figure 9 The denoising system used in this work

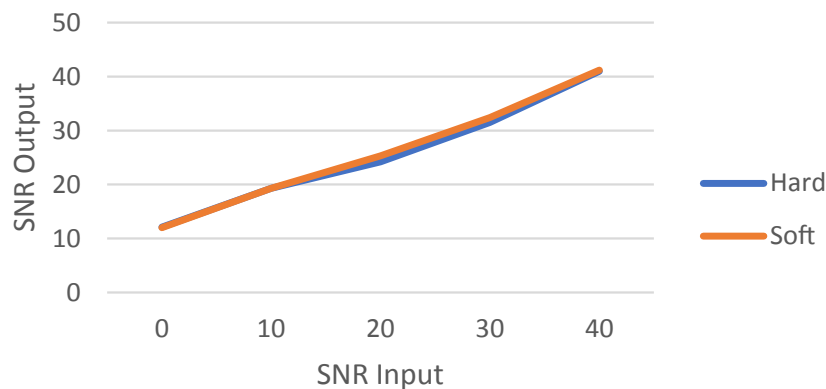


Figure 10 The performance of hard and soft threshold functions through different SNR input values

For testing the obtained optimal denoising parameters, 10 records (100 m, 101 m, 102 m, 103 m, 104 m, 200 m, 201 m, 202 m, 203 m, and 205 m) have been used from MIT-BIH Arrhythmia Database [28] and the results are shown in Figure 11 using 10 db as a signal to noise input ratio, soft as threshold function, rigsure as threshold value, db5 as wavelet filter, 7 levels of decomposition, and one as scale factor.

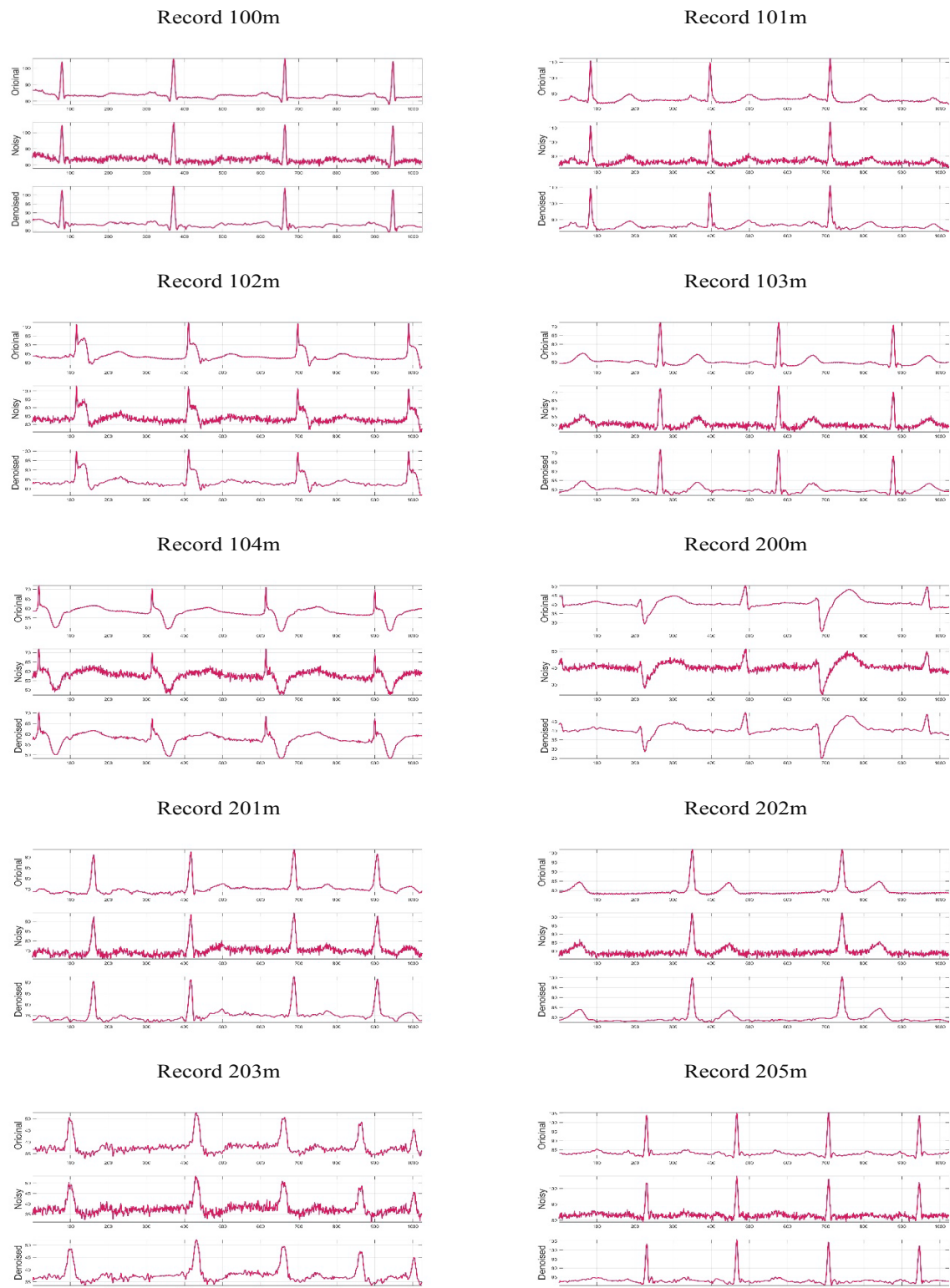


Figure 11 The denoising of different ECG Arrhythmia record signals using 10 dB

The results of this work are compared with the results of other researches in the same field. The first is done with the results of Zaid, Mohammed, Ahmad, and Laith's paper [13] who worked on ECG signal denoising based on wavelet transform and hill climbing algorithm, the second is done with those of El-Dahshan [5] who worked on wavelet

denoising on corrupted ECG signals with AWGN, the last is done with those of Chitrangi, and Harishchandra's [12], as illustrated in the Figure 12.

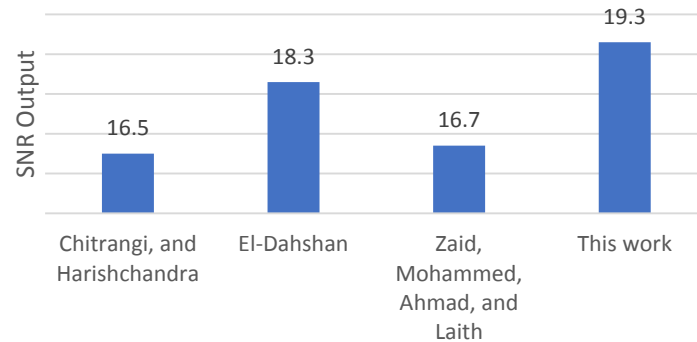


Figure 12 The SNR output performance of this work compared with others

CONCLUSION

In this work, WT has been applied with GA and proposed a method to guide the selection of the main WT denoising parameters to denoise ECG Arrhythmia signals corrupted by AWGN.

1. The soft threshold function is better than hard threshold function in applying the threshold value, it is illustrated in the Figure 11.
2. Merging GA with WT is a good technique for noise removal, because the denoising results of the proposed system on the ECG signal was good, this implies that the proposed method can denoise with different input SNR values..

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